



Homework Assignment 2, Due Feb 20, 2007

Your answers must be justified by showing all work. For homework problems requiring work in Matlab, you must submit your copy of the commented matlab code as well as an original (no photocopies) of any figures, images, etc. For homework problems requiring written explanations and/or discussions, all answers must be in your own words. That is, DO NOT copy from the text book, lecture notes, websites, etc. Answers to all homework problems (except Matlab code and figures generated in Matlab) must be handwritten.

- 1) There are many properties of the DTFT. You should have derived and examined these in a 1-D DSP course. Now consider extending these to the 2-D case. Complete the chart below:

$$\begin{array}{ll}
 f(x, y) & \Leftrightarrow F(u, v) \\
 f_1(x) f_2(y) & \Leftrightarrow \\
 f(x - X, y - Y) & \Leftrightarrow \\
 f(x, y) h(x, y) & \Leftrightarrow \\
 f(x, y) ** h(x, y) & \Leftrightarrow \\
 f(ax, by) & \Leftrightarrow \\
 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |f(x, y)|^2 dx dy & \Leftrightarrow
 \end{array}$$

Now, use these properties to find the Fourier transform of the following,

$$\begin{array}{ll}
 \cos(x) \delta(y) & \Leftrightarrow \\
 4 \delta(x + 3, y) & \Leftrightarrow \\
 5 \cos\left(\frac{30\pi x}{5}\right) e^{j20\pi y} & \Leftrightarrow
 \end{array}$$

- 2) Hand compute the 4-point DFT of the following 2-D signal:

$$\begin{bmatrix}
 1 & 1 & 2 & 2 \\
 1 & 1 & 2 & 2 \\
 2 & 2 & 4 & 4 \\
 2 & 2 & 4 & 4
 \end{bmatrix}$$

Verify your answers using Matlab (fft2).

- 3) For the following 4x4 digital image, hand calculate the 4x4 Haar transform:

$$\begin{bmatrix}
 1 & 1 & 2 & 2 \\
 1 & 1 & 2 & 2 \\
 4 & 4 & 16 & 4 \\
 1 & 1 & 1 & 1
 \end{bmatrix}$$

- Find the transformation matrix, and compute the transform using matrix multiplies.
- Using the rows of the transformation matrix, compute the basis images. Then compute the transform using basis images.
- Repeat (a) for the Hadamard transform.
- Check your answers to (a) and (c) using Matlab.

- 4) Consider the 4x4 image provided in problem 2. Compute the DCT of the image using “DCT” in MATLAB, you may use “dct2” only to check your results. Explain how you used the one-dimensional DCT Matlab function to compute a two-dimensional transform.
- 5) We have 25 hyperspectral image cubes (obtained via a satellite remote sensing system), where each is 256x256x100. We want to use the Principal Component Transform to reduce the data cube down to 256x256x10. Draw a block diagram showing the steps of this process.
- 6) Consider the following discrete signal (a row of pixels from a 2D image):

$$[1, 1, 4, 4, 1, 1, 2, 2, 4, -4, 16, 4, 1, 1, 1, 1],$$

and the Haar mother wavelet:

$$\frac{1}{\sqrt{2}}[1, -1].$$

- a) Sketch the signal.
- b) Hand calculate and sketch the discrete (dyadic) wavelet transform (DWT) of the signal using the “inner product definition” of the DWT. That is, each wavelet coefficient is calculated via an inner product of the signal and a scaled-shifted mother wavelet. Compute the DWT for scales 1,2, and 4.
- c) Now, hand calculate the DWT of the signal using the fast-algorithm, the dyadic filter tree with down sampling, where the lowpass and highpass filter impulse responses for the Haar mother wavelet are

$$\bar{h}_{lowpass} = \frac{1}{\sqrt{2}}[1, 1]$$

$$\bar{h}_{highpass} = \frac{1}{\sqrt{2}}[-1, 1]$$

Compute the wavelet decomposition for 2 levels of the filter tree, resulting in detail coefficients at level 1 and 2 and approximation coefficients at level 2. How do they compare to the results of part (b)? (You may use Matlab to compute your 1D convolutions, so as not to have to do them all out by hand.)

- d) Use the function “WAVEDEC” in Matlab to check your answers to part (b) and (c).
- 7) Upload the image “triangle.tif” from the course website into Matlab. Compute the 2D DFT (using FFT2 function) of the image. Display the magnitude response (using abs function) two times, once using the fftshift function and once without using the fftshift function. Explain the results that you’ve obtained. For example, where is the DC component and why is it located where it is? Which Fourier components are dominant and why?