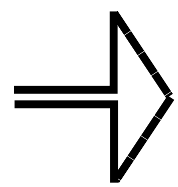
ECE 3323
ELECTROMAGNETICS II
HOMEWORK - ANTENNAS

ANT-1. A Hertzian dipole of length Δl = 0.6 mm in air carries a current of I_o = 10 mA at a frequency of 10 GHz. The antenna is located at the coordinate origin and the antenna current is oriented along the z-axis. Determine (a.) the electrical length of the antenna (b.) the far field phasor electric and magnetic field magnitudes at the spherical coordinate point P defined by ($R = 20\lambda$, $\theta = 90°$, $\varphi = 90°$) (c.) the time-average vector power density at the point P (d.) the total power radiated by the antenna (e.) the radiation resistance of the antenna. [Answer: (a.) $\Delta l = \lambda/50$ (b.) $|\tilde{E}_\theta| = 62.8$ mV/m, $|\tilde{H}_\varphi| = 167$ μA/m (c.) $\boldsymbol{P}_{ave} = 5.23\,\hat{\boldsymbol{R}}$ μW/m^2 (d.) 15.8 μW (e.) 0.316 Ω]

ANT-2. The current distribution on a center-fed short dipole (length $l \ll \lambda$) centered at the coordinate origin and lying along the z-axis can be written as

$$\tilde{\boldsymbol{I}}(z') = \begin{Bmatrix} I_o\left(1 - \dfrac{2z}{l}\right)\hat{\boldsymbol{z}} & (0 \le z' \le l/2) \\ I_o\left(1 + \dfrac{2z}{l}\right)\hat{\boldsymbol{z}} & (-l/2 \le z' \le 0) \end{Bmatrix}$$



where I_o defines the peak value of the current at the feed point of the antenna. Show that the magnetic vector potential of the center-fed short dipole is related to the magnetic vector potential of the Hertzian dipole by

$$\boldsymbol{A}_{short\ dipole} = \frac{1}{2}\boldsymbol{A}_{Hertzian\ dipole}$$

given equal antenna lengths and assuming the value of I_o is equal on the two antennas.

ANT-3. Based on the results from ANT-2, determine expressions for the following quantities: (a.) the short dipole far fields (b.) the time-average power density in the far field of the short dipole (c.) the total radiated power radiated by the short dipole (d.) the radiation resistance of the short dipole.

[Answer: (a.) $\tilde{\boldsymbol{E}} = \dfrac{j\eta_o k I_o l}{8\pi R}\sin\theta\, e^{-jkR}\,\hat{\boldsymbol{\theta}}$, $\tilde{\boldsymbol{H}} = \dfrac{jkI_o l}{8\pi R}\sin\theta\, e^{-jkR}\,\hat{\boldsymbol{\varphi}}$

(b.) $\boldsymbol{P}_{ave} = \dfrac{\eta_o}{2}\left(\dfrac{kI_o l}{8\pi R}\sin\theta\right)^2\hat{\boldsymbol{R}}$ (c.) $P_{rad} = 10\pi^2\left(I_o\dfrac{l}{\lambda}\right)^2$

(d.) $R_{rad} = 20\pi^2\left(\dfrac{l}{\lambda}\right)^2$]

ANT-4. Determine the efficiency of a short dipole antenna of length l = 5 cm made of 12 gauge (a = 1.027mm) aluminum wire when operated at (a.) 150 MHz (b.) 300 MHz. [Answer: (a.) ξ = 88.59% (b.) ξ =95.64%]

ANT-5. A copper wire dipole ($l = 300$ mm, $a = 1$ mm) is operated at 100 MHz and radiates 0.5 W. Determine (a.) the required peak antenna current (b.) the amount of power dissipated by the antenna (c.) the power density at a distance of 20 m in the direction of maximum radiation (d.) the radiation efficiency. [Answer: (a.) $I_o = 712$ mA (b.) $P_{loss} = 15.8$ mW (c.) $P_{ave} = 149\ \mu\text{W/m}^2$ (d.) $\xi = 96.9\%$]

ANT-6. An antenna in air radiates the following far field electric field.

$$\tilde{\boldsymbol{E}} = E_\theta \hat{\boldsymbol{\theta}} = \frac{5}{R} \sin 2\theta \, e^{-jkR} \, \hat{\boldsymbol{\theta}} \ \ (\text{V/m})$$

Determine the total power radiated by this antenna. [Answer: $P_{rad} = 222$ mW]

ANT-7. Determine the peak current and total radiated power required by

(a.) a Hertzian dipole of length $l = \lambda/25$

(b.) a half wave dipole

in order to produce a magnetic field of 5 μA/m in the direction of maximum radiation at a distance of 2 km. [Answer: (a.) $I_o = 500$ mA, $P_{rad} = 158$ mW (b.) $I_o = 62.8$ mA, $P_{rad} =$ 144 mW]

ANT-8. Assuming a lossless dipole ($R_{loss} = 0$) connected to a 75 Ω lossless transmission line, determine the reflection coefficient, the standing wave ratio, and the percentage of the incident power radiated by the antenna if the dipole impedance is (a.) $Z_A = (73 + j42.5)$ Ω (a true half wave dipole, $l = \lambda/2$) (b.) $Z_A = (72 + j0)$ Ω (a resonant half wave dipole trimmed to a length slightly less than $l = \lambda/2$).
[Answer: (a.) $\Gamma = 0.0637 + j0.2689$, $s = 1.764$, 92.37% (b.) $\Gamma = -0.0204 + j0$, $s = 1.042$, 99.96%]

ANT-9. A copper wire loop antenna (loop radius = 4 cm, wire radius = 1.5 mm) in air lying in the x-y plane and centered at the coordinate origin operates at 200 MHz. Determine (a.) the loop radiation resistance (b.) the loop loss resistance (c.) the loop radiation efficiency (d.) the current required to produce a power density of 1 $\mu\text{W/m}^2$ at a distance of 50 m in the direction of maximum radiation. [Answer: (a.) $R_{rad} = 156$ mΩ (b.) $R_{loss} = 98.4$ mΩ (c.) $\xi = 61.3\%$ (d.) $I_o = 519$ mA]

ANT-10. The far field power density of a particular antenna is given by

$$\boldsymbol{P}_{ave} = 9 I_o^2 \, \frac{\sin^2\theta \cos^2\varphi}{R^2} \, \hat{\boldsymbol{R}} \quad (\text{W/m}^2)$$

where I_o is the peak current on the antenna. Determine the radiation resistance of this antenna. [Answer: $R_{rad} = 75.4$ Ω]

ANT-11. For the antenna of ANT-10, determine (a.) the direction(s) of maximum radiation (b.) the radiation intensity function (c.) directivity in dB (d.) the beam solid angle (e.) the HPBW in the $\varphi = 0°$ plane. [Answer: (a.) $\theta = \pi/2$, $\varphi = 0$ ($+x$ axis) and $\theta = \pi/2$, $\varphi = \pi$ ($-x$ axis) (b.) $U(\theta,\varphi) = 9I_o^2\sin^2\theta\cos^2\varphi$ W (c.) $D = 4.77$ dB (d.) $\Omega_A = 4\pi/3$ sr (e.) HPBW = 90°]

ANT-12. An antenna with a beam solid angle of 1.4 sr radiates a total power of 50 W. Determine the maximum power density in the far field of the antenna at a distance of 3 km. [Answer: 3.97 $\mu W/m^2$]

ANT-13. If the antenna defined in ANT-10 has a loss resistance of 2 Ω, determine the gain of the antenna in dB. [Answer: 4.66 dB]

ANT-14. A three element linear array of small loop antennas (radii = 5 cm, loop axes lie along the z-axis) operates at 30 MHz. The element spacing is $d = 0.5\lambda$ and the element are driven in-phase ($\alpha = 0$). (a.) Determine an expression for the far field electric field produced by the array (b.) Plot the power pattern for the antenna array.

$$\left[\text{Answer:} \quad \text{(a.)}\ \tilde{E}_\theta = \frac{\eta_o k^2 I_o \Delta S}{4\pi R} e^{-jkR} \sin\theta \frac{\sin\left(\frac{3\pi}{2}\cos\theta\right)}{\sin\left(\frac{\pi}{2}\cos\theta\right)} \right]$$

ANT-15. For the array given in ANT-14, (a.) determine the phase shift α required to produce an array factor maximum at an elevation angle of $\theta = 75°$ (b.) plot the power pattern of this antenna array. [Answer: (a.) $-46.58°$]

ANT-16. Determine the following for a lossless half-wave dipole antenna. (a.) Show that the antenna directivity is 2.15 dB (hint: use the numerical integration results from the lossless half-wave dipole radiation resistance calculation) (b.) Determine the antenna effective area at 100 MHz. (c.) Determine the antenna effective area at 300 MHz. [Answer: (b.) 1.18 m^2 (c.) 0.131 m^2]

ANT-17. Two identical antennas separated by 20 m are utilized as transmit and receive antennas in a 10 GHz system. The cable loss between the transmitter and the transmit antenna is 3.2 dB while the cable loss between the receive antenna and the receiver is 4.6 dB. If the ratio of transmitted power (P_t) to received power (P_r) is -40 dB, determine the gain of the antennas in dB. [Answer: 23.1 dB]

ANT-18. A communications link operating at 420 MHz utilizes half-wave dipoles separated by a distance of 5 km. Neglecting antenna and cable losses, determine the required transmitted power if the system receiver is characterized by a bandwidth of 4 MHz, a system noise temperature of 600 K, and a required signal-to-noise ratio of 20 dB. [Answer: 9.53 mW]

ANT-19. A monostatic X-band radar (f = 10 GHz) transmits 10 kW utilizing a single transmit/receive antenna with a gain of 35 dB. The radar tracks a target with an RCS of 3 m^2 at a range of 2 km. The radar receiver is characterized by a noise temperature of 580 K and a received signal bandwidth of 5 MHz. Determine (a.) the incident electric field magnitude at the target (b.) the scattered electric field magnitude at the receive antenna (c.) the received power (d.) the signal-to-noise ratio of the radar receiver. [Answer: (a.) 21.78 V/m (b.) 5.32 mV/m (c.) 8.50 nW (d.) 53.3 dB]