Loop Antennas

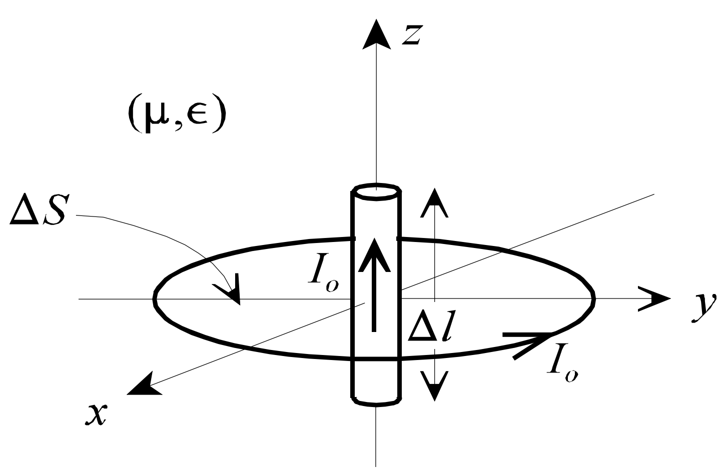
Loop antennas have the same desirable characteristics as dipoles and monopoles in that they are inexpensive and simple to construct. Loop antennas come in a variety of shapes (circular, rectangular, elliptical, etc.) but the fundamental characteristics of the loop antenna radiation pattern (far field) are largely independent of the loop shape.

Just as the electrical length of the dipoles and monopoles effect the efficiency of these antennas, the electrical size of the loop (circumference) determines the efficiency of the loop antenna. Loop antennas are usually classified as either electrically small or electrically large based on the circumference of the loop.

electrically small loop $\Rightarrow$ circumference $\leq \lambda/10$

electrically large loop $\Rightarrow$ circumference $\approx \lambda$

The electrically small loop antenna is the dual antenna to the electrically short dipole antenna when oriented as shown below. That is, the far-field electric field of a small loop antenna is identical to the far-field magnetic field of the short dipole antenna and the far-field magnetic field of a small loop antenna is identical to the far-field electric field of the short dipole antenna.



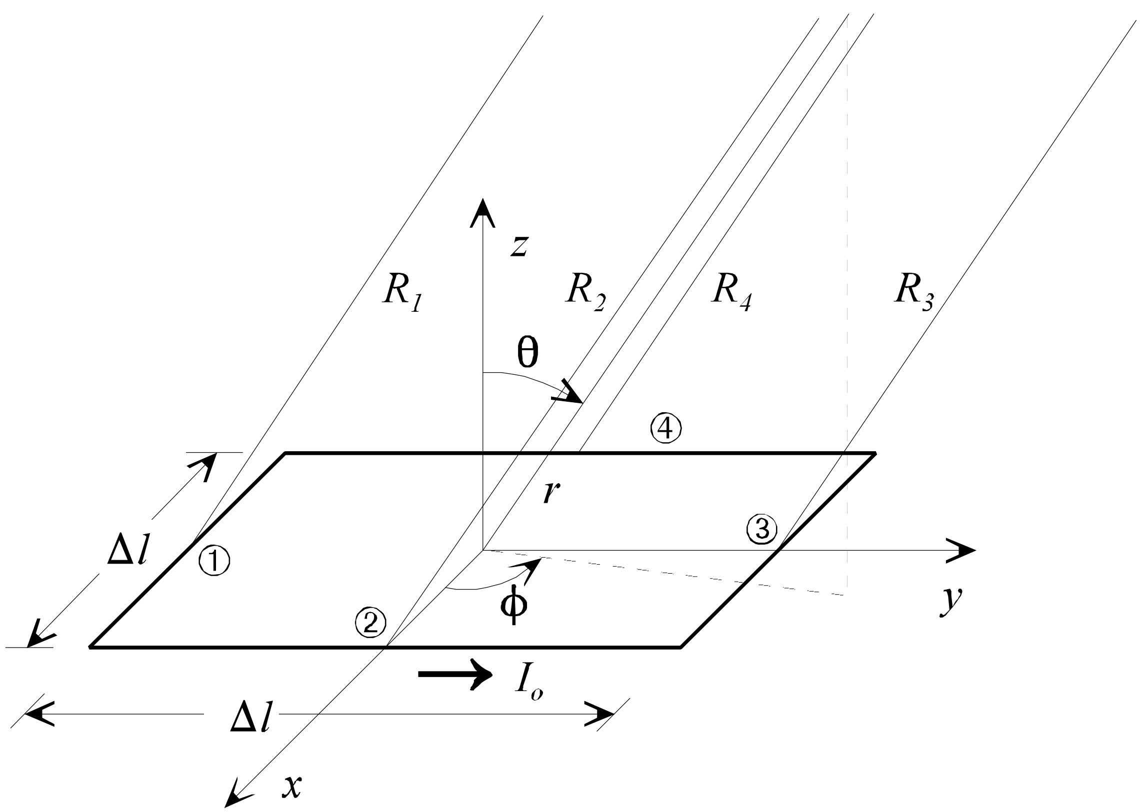
$$E_{loop} = H_{dipole}$$
$$H_{loop} = E_{dipole}$$
(far fields)

Given that the radiated fields of the short dipole and small loop antennas are dual quantities, the radiated power for both antennas is the same and therefore, the radiation patterns are the same. This means that the plane of maximum radiation for the loop is in the plane of the loop. When operated as a receiving antenna, we know that the short dipole must be oriented such that the electric field is parallel to the wire for maximum response. Using the concept of duality, we find that the small loop must be oriented such that the magnetic field is perpendicular to the loop for maximum response.

The radiation resistance of the small loop is much smaller than that of the short dipole. The loss resistance of the small loop antenna is frequently much larger than the radiation resistance. Therefore, the small loop antenna is rarely used as a transmit antenna due to its extremely small radiation efficiency. However, the small loop antenna is acceptable as a receive antenna since signal-to-noise ratio is the driving factor, not antenna efficiency. The fact that a significant portion of the received signal is lost to heat is not of consequence as long as the antenna provides a large enough signal-to-noise ratio for the given receiver. Small loop antennas are frequently used for receiving applications such as pagers, low-frequency portable radios, and direction finding. Small loops can also be used at higher frequencies as field probes providing a voltage at the loop terminals which is proportional to the field passing through the loop.

Electrically Small Loop Antenna

The far fields of an electrically small loop antenna are dependent on the loop area but are independent of the loop shape. Since the magnetic vector potential integrations required for a circular loop are more complex than those for a square loop, the square loop is considered in the derivation of the far fields of an electrically small loop antenna. The square loop, located in the *x*-*y* plane and centered at the coordinate origin, is assumed to have an area of Δl^2 and carry a uniform current I_o.



The square loop may be viewed as four segments which each represent an infinitesimal dipole carrying current in a different direction. In the far field, the distance vectors from the centers of the four segments become almost parallel.

$$R_1 \approx r + \frac{\Delta l}{2}\sin\theta\sin\phi$$

$$R_2 \approx r - \frac{\Delta l}{2}\sin\theta\cos\phi$$

$$R_3 \approx r - \frac{\Delta l}{2}\sin\theta\sin\phi$$

$$R_4 \approx r + \frac{\Delta l}{2}\sin\theta\cos\phi$$

As always in far field expressions, the above approximations are used in the phase terms of the magnetic vector potential, but we may assume that $R_1 \approx R_2 \approx R_3 \approx R_4 \approx r$ for the magnitude terms. The far field magnetic vector potential of a *z*-directed infinitesimal dipole centered at the origin is

$$\boldsymbol{A} \approx \frac{\mu I_o \Delta l}{4\pi r} e^{-jkr}\,\boldsymbol{a}_z$$

The individual far field magnetic vector potential contributions due to the four segments of the current loop are

$$\boldsymbol{A_1} \approx \frac{\mu I_o \Delta l}{4\pi r} e^{-jk\left(r + \frac{\Delta l}{2}\sin\theta\sin\phi\right)}\boldsymbol{a}_x$$

$$\boldsymbol{A_2} \approx \frac{\mu I_o \Delta l}{4\pi r} e^{-jk\left(r - \frac{\Delta l}{2}\sin\theta\cos\phi\right)}\boldsymbol{a}_y$$

$$\boldsymbol{A_3} \approx \frac{\mu I_o \Delta l}{4\pi r} e^{-jk\left(r - \frac{\Delta l}{2}\sin\theta\sin\phi\right)}(-\boldsymbol{a}_x)$$

$$\boldsymbol{A_4} \approx \frac{\mu I_o \Delta l}{4\pi r} e^{-jk\left(r + \frac{\Delta l}{2}\sin\theta\cos\phi\right)}(-\boldsymbol{a}_y)$$

Combining the *x*-directed and *y*-directed terms yields

$$\mathbf{A_1} + \mathbf{A_3} \approx \frac{\mu I_o \Delta l}{4\pi r} e^{-jkr} \left[e^{-jk\frac{\Delta l}{2}\sin\theta\sin\phi} - e^{jk\frac{\Delta l}{2}\sin\theta\sin\phi} \right] \mathbf{a}_x$$

$$= -j \frac{\mu I_o \Delta l}{2\pi r} e^{-jkr} \sin\left(\frac{k\Delta l}{2} \sin\theta\sin\phi \right) \mathbf{a}_x$$

$$\mathbf{A_2} + \mathbf{A_4} \approx \frac{\mu I_o \Delta l}{4\pi r} e^{-jkr} \left[e^{jk\frac{\Delta l}{2}\sin\theta\cos\phi} - e^{-jk\frac{\Delta l}{2}\sin\theta\cos\phi} \right] \mathbf{a}_y$$

$$= j \frac{\mu I_o \Delta l}{2\pi r} e^{-jkr} \sin\left(\frac{k\Delta l}{2} \sin\theta\cos\phi \right) \mathbf{a}_y$$

For an electrically small loop ($\Delta l << \lambda$), the arguments of the sine functions above are very small and may be approximated according to

$$\sin(x) \approx x \qquad\qquad x \ll 1$$

which gives

$$\mathbf{A_1} + \mathbf{A_3} \approx -j \frac{k\mu I_o \Delta l^2}{4\pi r} e^{-jkr} \sin\theta\sin\phi\, \mathbf{a}_x$$

$$\mathbf{A_2} + \mathbf{A_4} \approx j \frac{k\mu I_o \Delta l^2}{4\pi r} e^{-jkr} \sin\theta\cos\phi\, \mathbf{a}_y$$

The overall vector potential becomes

$$\mathbf{A} = \mathbf{A_1} + \mathbf{A_2} + \mathbf{A_3} + \mathbf{A_4}$$

$$\approx j \frac{k\mu I_o \Delta S}{4\pi r} e^{-jkr} \sin\theta \left(-\sin\phi\, \mathbf{a}_x + \cos\phi\, \mathbf{a}_y \right)$$

where $\Delta S = \Delta l^2$ = loop area. The bracketed term above is the spherical coordinate unit vector $\mathbf{a}_\phi$.

$$\mathbf{A} \approx j \frac{k\mu I_o \Delta S}{4\pi r} e^{-jkr} \sin\theta\, \mathbf{a}_\phi$$

Electrically small current loop far field magnetic vector potential

The corresponding far fields are

$$E_\phi \approx -j\omega A_\phi = \frac{\omega k \mu I_o \Delta S}{4\pi r} e^{-jkr} \sin\theta$$

$$H_\theta \approx j\frac{\omega}{\eta} A_\phi = -\frac{\omega k \mu I_o \Delta S}{\eta 4\pi r} e^{-jkr} \sin\theta$$

$$\omega k \mu = (\omega \mu)\, \omega\sqrt{\mu\epsilon} = \omega^2 \mu\epsilon \sqrt{\frac{\mu}{\epsilon}} = \eta k^2$$

$$E_\phi \approx \frac{\eta k^2 I_o \Delta S}{4\pi r} e^{-jkr} \sin\theta$$

$$H_\theta \approx = -\frac{k^2 I_o \Delta S}{4\pi r} e^{-jkr} \sin\theta$$

Electrically small current loop far fields

The fields radiated by an electrically small loop antenna can be increased by adding multiple turns. For the far fields, the added height of multiple turns is immaterial and the resulting far fields for a multiple turn loop antenna can be found by simply multiplying the single turn loop antenna fields by the number of turns *N*.

$$E_\phi \approx \frac{\eta k^2 N I_o \Delta S}{4\pi r} e^{-jkr} \sin\theta$$

$$H_\theta \approx = -\frac{k^2 N I_o \Delta S}{4\pi r} e^{-jkr} \sin\theta$$

Electrically small multiple turn current loop far fields

Dual and Equivalent Sources

(Electric and Magnetic Dipoles and Loops)

If we compare the far fields of the infinitesimal dipole and the electrically small current loop with electric and magnetic currents, we find pairs of equivalent sources and dual sources.

Infinitesimal electric dipole

$$E_\theta \approx j\frac{\eta k I_o \Delta l}{4\pi r}\, e^{-jkr}\,\sin\theta$$

$$H_\phi \approx = j\frac{k I_o \Delta l}{4\pi r}\, e^{-jkr}\,\sin\theta$$

Small electric current loop

$$E_\phi \approx \frac{\eta k^2 I_o \Delta S}{4\pi r}\, e^{-jkr}\,\sin\theta$$

$$H_\theta \approx = -\frac{k^2 I_o \Delta S}{4\pi r}\, e^{-jkr}\,\sin\theta$$

Using duality, we may determine the far fields of the corresponding magnetic geometries.

Electric source	Magnetic source
$\boldsymbol{E}$	$\boldsymbol{H}$
$\boldsymbol{H}$	$-\boldsymbol{E}$
$\boldsymbol{I_o}$	$\boldsymbol{I_{om}}$
k	k
η	$1/\eta$

Infinitesimal magnetic dipole

$$H_\theta \approx j\frac{k I_{om} \Delta l}{\eta 4\pi r}\, e^{-jkr}\,\sin\theta$$

$$E_\phi \approx -j\frac{k I_{om} \Delta l}{4\pi r}\, e^{-jkr}\,\sin\theta$$

Small magnetic current loop

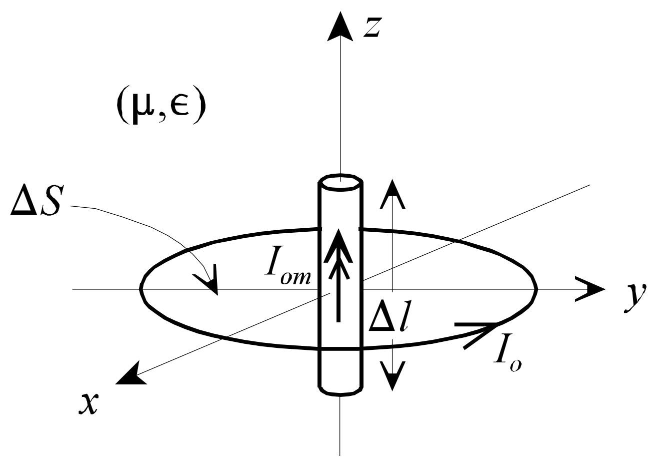
$$H_\phi \approx \frac{k^2 I_{om} \Delta S}{\eta 4\pi r}\, e^{-jkr}\,\sin\theta$$

$$E_\theta \approx = \frac{k^2 I_{om} \Delta S}{4\pi r}\, e^{-jkr}\,\sin\theta$$

If, for the small electric current loop and the infinitesimal magnetic dipole, we choose

$$I_{om}\Delta l = j\eta k I_o \Delta S = j\omega\mu I_o \Delta S$$

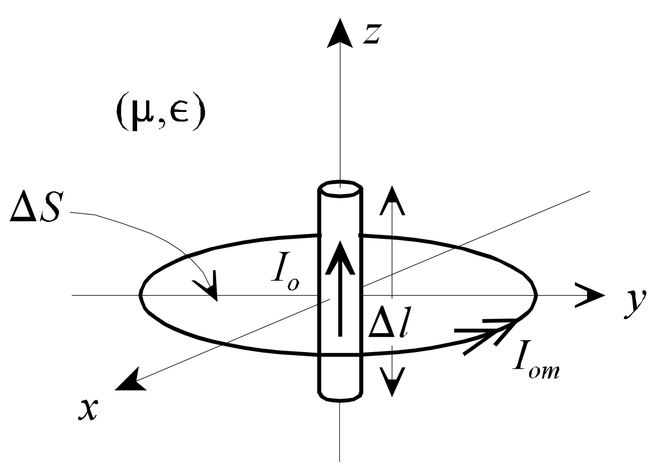
then the far fields radiated by these two sources are identical (the small electric current loop and the infinitesimal magnetic dipole are equivalent sources).



Similarly, for the small magnetic current loop and the infinitesimal electric dipole, if we choose

$$I_o\Delta l = j\eta k I_{om} \Delta S = j\omega\mu I_{om} \Delta S$$

then the far fields radiated by these two sources are identical (the small magnetic current loop and the infinitesimal electric dipole are *equivalent sources*).



The infinitesimal electric and magnetic dipoles are defined as *dual sources* since the magnetic field of one is identical to the electric field of the other when the currents and dimensions are chosen appropriately. Likewise, the small electric and magnetic current loops are *dual sources*.

We also find from this discussion of dual and equivalent sources that the polarization of the far fields for the dual sources are orthogonal. In the plane of maximum radiation (*x*-*y* plane), the four sources have the following far field polarizations

infinitesimal electric dipole	⇒	vertical polarization
infinitesimal magnetic dipole	⇒	horizontal polarization
small electric current loop	⇒	horizontal polarization
small magnetic current loop	⇒	vertical polarization

Loop Antenna Characteristics

The time-average Poynting vector in the far field of the multiple-turn electrically small loop is

$$\boldsymbol{S} = \frac{1}{2}\boldsymbol{E}\times\boldsymbol{H}^* = \frac{1}{2}\left[E_\phi\,\boldsymbol{a}_\phi \times H_\theta^*\,\boldsymbol{a}_\theta\right]$$

$$= \frac{1}{2}\left[E_\phi\,\boldsymbol{a}_\phi \times \frac{-E_\phi^*}{\eta}\,\boldsymbol{a}_\theta\right] = \frac{|E_\phi|^2}{2\eta}\,\boldsymbol{a}_r$$

$$= \frac{\eta}{2}\left[\frac{k^2 N |I_o| \Delta S}{4\pi r}\sin\theta\right]^2 \boldsymbol{a}_r$$

$$= \frac{\eta}{2}\left[\frac{\pi N \Delta S}{\lambda^2 r}\right]^2 |I_o|^2 \sin^2\theta\, \boldsymbol{a}_r$$

The radiation intensity function is

$$U(\theta,\phi) = r^2 S = \frac{\eta}{2}\left[\frac{\pi N \Delta S}{\lambda^2}\right]^2 |I_o|^2 \sin^2\theta$$

Loop antenna radiation intensity function

The maximum value of the radiation intensity function is

$$[U(\theta,\phi)]_{\max} = \frac{\eta}{2}\left[\frac{\pi N \Delta S}{\lambda^2}\right]^2 |I_o|^2$$

The radiated power is

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi = \frac{\eta}{2}\left[\frac{\pi N \Delta S}{\lambda^2}\right]^2 |I_o|^2 (2\pi)\int_0^{\pi}\sin^3\theta\, d\theta$$

$$P_{rad} = \frac{4}{3}\eta\pi^3\left[\frac{N\Delta S}{\lambda^2}\right]^2 |I_o|^2$$

Loop antenna radiated power

The radiation resistance of the loop antenna is found from the radiated power.

$$P_{rad} = \frac{4}{3}\eta\pi^3\left[\frac{N\Delta S}{\lambda^2}\right]^2 |I_o|^2 = \frac{1}{2}|I_o|^2 R_r$$

$$R_r = \frac{8}{3}\eta\pi^3\left[\frac{N\Delta S}{\lambda^2}\right]^2$$

$$R_r = 320\,\pi^4\left[\frac{N\Delta S}{\lambda^2}\right]^2 = 31{,}171\,\frac{N^2\Delta S^2}{\lambda^4}$$

Loop antenna in air radiation resistance

The directivity of the loop antenna is defined by

$$D(\theta,\phi) = 4\pi\frac{U(\theta,\phi)}{P_{rad}} = 4\pi\frac{\dfrac{\eta}{2}\left[\dfrac{\pi N\Delta S}{\lambda^2}\right]^2 |I_o|^2\sin^2\theta}{\dfrac{4}{3}\eta\pi^3\left[\dfrac{N\Delta S}{\lambda^2}\right]^2 |I_o|^2}$$

$$D(\theta,\phi) = 1.5\sin^2\theta$$

Loop antenna directivity function

Given the same directivity function as the infinitesimal dipole, the loop antenna has the same maximum directivity, effective aperture and beam solid angle as the infinitesimal dipole.

$$D_o = 1.5$$

$$A_e = \frac{\lambda^2}{4\pi}D_o = \frac{3\lambda^2}{8\pi}$$

$$\Omega_A = \frac{4\pi}{D_o} = \frac{8\pi}{3}$$

Loop antenna maximum directivity, effective aperture, and beam solid angle

If we compare the radiation resistances of the electrically short dipole and the electrically small loop (both antennas in air), we find that the radiation resistance of the small loop decreases much faster than that of the short dipole with decreasing frequency since

R_r (short dipole) ~ λ^{-2}

$$R_r = 20\,\pi^2 \left(\frac{l}{\lambda} \right)^2$$

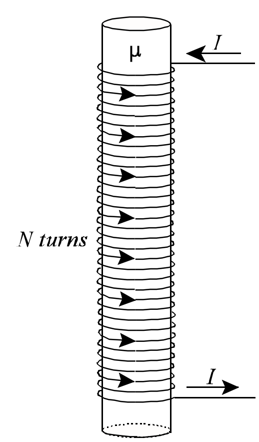
R_r (small loop) ~ λ^{-4}

$$R_r = 320\,\pi^4 \left[\frac{N \Delta S}{\lambda^2} \right]^2$$

The radiation resistance of the small loop can be increased significantly by adding multiple turns ($R_r \sim N^2$). However, the addition of more conductor length also increases the antenna loss resistance which reduces the overall antenna efficiency. To increase the radiation resistance without significantly reducing the antenna efficiency, the number of turns can be decreased when a ferrite material is used as the core of the winding. The general radiation resistance formula for a small loop with any material as its core is

$$R_r = \frac{8}{3}\,\eta\,\pi^3 \left[\frac{N \Delta S}{\lambda^2} \right]^2$$

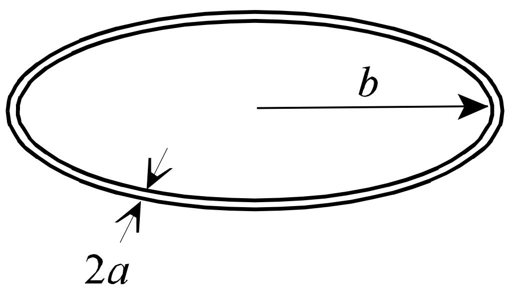
A multiturn loop which is wound on a linear ferrite core is commonly referred to as a *loop-stick antenna*. The loop-stick antenna is commonly used as a low-frequency receiving antenna.



Loop-stick antenna

Impedance of Electrically Small Antennas

The current density was assumed to be uniform on the electrically small current loop for our far field calculations. For a circular loop, the assumption of uniform current is accurate up to a loop circumference of about 0.2λ.



b = loop radius

a = wire radius

The restriction on the size of the constant current loop in terms of the loop radius is

$$2\pi b \leq 0.2\lambda \qquad \Rightarrow \qquad b \leq \frac{0.2}{2\pi}\lambda = 0.032\lambda$$

The electrically small current loop was found to be a dual source to the infinitesimal dipole. If we investigate the reactance of these dual electrically small antennas, we find that the dipole is capacitive while the loop is inductive. The exact reactance of the current loop is dependent on the shape of the loop. Approximate formulas for the reactance are given below for a short dipole and an electrically small circular current loop.

Infinitesimal Dipole (length = Δl, wire radius = a)

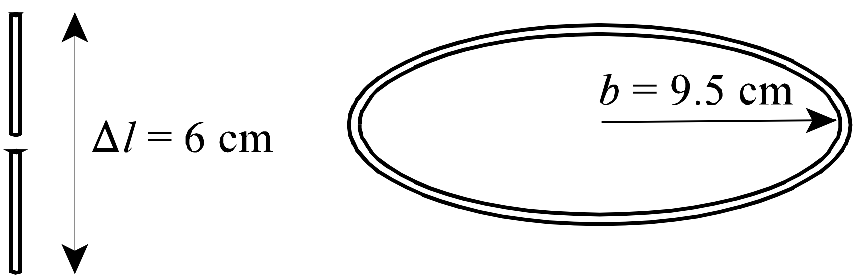
$$C \approx \frac{\pi \epsilon_o \Delta l}{\left[\ln\left(\frac{\Delta l}{a}\right) - 1\right]} \qquad X_A = -\frac{1}{\omega C} \approx -\frac{\left[\ln\left(\frac{\Delta l}{a}\right) - 1\right]}{\omega \pi \epsilon_o \Delta l}$$

Electrically Small Circular Current Loop (loop radius = *b*, wire radius = *a*)

$$L \approx \mu_o b \left[\ln\left(\frac{8b}{a} \right) - 2 \right] \qquad X_A = \omega L \approx \omega \mu_o b \left[\ln\left(\frac{8b}{a} \right) - 2 \right]$$

Example (Impedances of electrically small antennas)

Determine the total impedance and radiation efficiency of the following electrically small antennas operating at 1, 10 and 100 MHz. Both antennas are constructed using #10 AWG copper wire (a = 2.59 mm, $\sigma = 5.8 \times 10^7$ ℧/m).



$$f = 1\,\text{MHz} \qquad \lambda = 300\,\text{m}$$
$$f = 10\,\text{MHz} \qquad \lambda = 30\,\text{m}$$
$$f = 100\,\text{MHz} \qquad \lambda = 3\,\text{m}$$

$$@ f = 100\,\text{MHz} \qquad \frac{\Delta l}{\lambda} = \frac{0.06}{3} = 0.02 \qquad \frac{b}{\lambda} = \frac{0.095}{3} = 0.032$$

$$(R_r)_{dipole} = 80\pi^2 \left(\frac{\Delta l}{\lambda} \right)^2 \qquad (R_r)_{loop} = 320\pi^4 \left(\frac{\Delta S}{\lambda^2} \right)^2$$

$$(R_L)_{dipole} = \left(\frac{\Delta l}{2a}\right)\sqrt{\frac{f\mu_o}{\pi\sigma}} \qquad (R_L)_{loop} = \left(\frac{\pi b}{a}\right)\sqrt{\frac{f\mu_o}{\pi\sigma}}$$

$$(X_A)_{dipole} = -\frac{\left[\ln\left(\frac{\Delta l}{a}\right) - 1\right]}{2\pi^2 f \epsilon_o \Delta l} \qquad (X_A)_{loop} = \omega\mu_o b\left[\ln\left(\frac{8b}{a}\right) - 2\right]$$

$$e_{cd} = \frac{R_r}{R_r + R_L}$$

Infinitesimal Dipole

f (MHz)	Δl	R_r	R_L	e_{cd}	jX_A
1	0.0002λ	31.6 μΩ	0.962 mΩ	3.18 %	−204 kΩ
10	0.002λ	3.16 mΩ	3.04 mΩ	51.0 %	−20.4 kΩ
100	0.02λ	0.316 Ω	9.62 mΩ	97.0 %	−2.04 kΩ

Small Loop

f (MHz)	b	R_r	R_L	e_{cd}	jX_A
1	0.00032λ	3.09 nΩ	9.57 mΩ	3.2×10^{-5} %	2.76 Ω
10	0.0032λ	30.9 μΩ	30.3 mΩ	0.102 %	27.6 Ω
100	0.032λ	0.309 Ω	95.7 mΩ	76.4%	276 Ω